Modelling Capital Guaranteed Note on Hang Seng Index

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Section A:
The Basic Information

Investment Objectives:

--Enable investors to achieve potential capital appreciation by participating in the positive performance of the Hang Seng Index.
--Provide investors with a guaranteed minimum return of their investment at maturity of the Funds.
1. Hang Seng 90% Capital Guaranteed Hong Kong Equity Fund (90% Fund)
2. Hang Seng 100% Capital Guaranteed Hong Kong Equity Fund (100% Fund)
3. Hang Seng 110% Capital Guaranteed Hong Kong Equity Fund (110% Fund)

--Additional Potential Upside: Participation to the monthly average positive growth of the Hang Seng index during the 4 years investment period.

How Do the Funds Works?

Each of the Funds is composed of two parts: *Guaranteed Capital* and *Upside Potential of the Hang Seng Index* which will be linked to the monthly performance of the Hang Seng Index in the 4-year investment period.

Guaranteed Capital + Upside Potential of the Hang Seng Index

Where,
Guaranteed Capital = Initial investment x Capital guaranteed level (X% =90%, 100%, 110%)

Upside Potential of the HSI = Initial investment x Participation rate x Monthly average positive growth % of HSI

Return Calculation at Maturity ----- Upside Potential of HSI
Let the initial index level at launch date t = 0 be I (0),
During the 4 years periods, there are 48 months, let n be the number of months n = 0 to 48.
By the end of each months, let the index level be I (n).

Let Initial Investment be P, Capital Guaranteed level X %, and Participation Rate be k
Let C be the Final monthly average positive growth % of HSI

$$C = \left[\Sigma_{n=1 \text{ to } 48} \{ \text{Max} (I(n)/I(0) - 1 , 0) \} \right] / 48$$

So, for the X % level Guaranteed Fund with participation rate k, the Final Payoff at maturity (at the end of 4 years)

Final Payoff = P * X% + P * k * C

Key Benefits

--In order to best fit investor's investment objective, there is a *choice of Capital Guaranteed Level of 90%, 100%, 110%* of initial investment over a period of 4 years.

--Investors may enjoy a *Participation Rate as high as 100%* to maximize potential upside pertaining to the growth of HSI.

--The 3 Funds offer different capital guaranteed levels with different levels of participation to the Hang Seng Index. They are designed to match investors' different risk appetites for investment in the Hong Kong equity market. The lower the capital guaranteed level, the higher the level of participation to the upside potential of the Hang Seng Index.

→ For 90% G. Fund, ranges of participation rate: 85% -100%

→ For 100% G. Fund, ranges of participation rate: 40% -55%

→ For 110% G. Fund, ranges of participation rate: 5% -15%

Fund Information:

Offer Price:	US $10
Launch Date:	1 March, 2002
Maturity Date:	3 March, 2006
Base Currency:	US $
Redemption Fee:	2%
Management Fee:	up to 1.25% p.a.
Guarantor:	Hang Seng Bank Limited

How to structure the Fund:

The Funds will invest in ***Hang Seng index-linked Notes*** whose return at maturity will match that of the Funds respectively. In effect, ***the Notes will comprise zero coupon instruments (4-year zero coupon bond) and Hang Seng Index-linked option. (Asian type Call option which take the 48-average monthly positive return as payoff)***.

Section B:
How to price the Hang Seng Capital Guaranteed Hong Kong Equity Fund?

Monte Carlo Simulation is used to price this product. This is the simplest method that can be used to price most of the exotic product. *Two important issues need to be considered are: 1) the asset model of the underlying index, 2) The payoff function of the fund at maturity.*

Asset model of the underlying index:

-- Constant volatility vs GARCHs model:

According to the classic Black-Scholes options pricing (B-S) model all options based on the same underlying sharing a constant implied volatility under the assumption of a geometric Brownian motion process. But if this model is used to back-test the market-traded option, we can observe that different contracts produce significantly different implied volatilities. Options' implied volatilities actually vary with the different time to maturity. This is referred to as the term structure of implied volatility. For a given time to maturity, implied volatilities for different strikes are not the same either. This is known as the implied volatility skew and often referred as the volatility smile. All these market evidences imply that the option market "expects" the future volatility of the underlying asset will not be a constant.

Facts: VIX in US market---implied volatility index:

One measure of the level of implied volatility in index options is CBOE's Volatility Index, VIX. VIX, introduced by CBOE in 1993, measures the volatility of the U.S. equity market. It provides investors with up-to-the-minute market estimates of expected volatility by using real-time OEX index option bid/ask quotes. This index is calculated by taking a weighted average of the implied volatilities of eight OEX calls and puts. *It is used by some traders as a general indication of index option implied volatility. Implied volatility levels in index options change frequently and substantially.* This Market Volatility Index is sometimes called "investor fear gauge".

<u>What is GARCHs</u> ?

A Stochastic volatility models is more reasonable for option pricing. It can explain the basic shapes of the smile patterns and allow for more realistic theories of the term structure of implied volatility. A particular case is that volatility can be described with a GARCH model. In GARCH models, the variance is written as a function of past returns, but with *exponentially smoothing and a certain time-decay factor*. One more important feature of GARCH is that the constant term in the recursive equation allows GARCH to capture the notion that the *volatility is mean reverting*, and allow the model to be used for forecasting volatility. Other alternative models such as the GJR-GARCH and NGARCH allow *negative returns to have more effect* on the volatility than positive returns.

--By comparing the simulation results under GARCH model and BS model, we can see that the BS model will over-price the Fund value in high volatility environment, but under-price the Fund value in low volatility environment. See **

GARCH, N-GARCH, GJR-GRACH ------ which one is better?

To estimate the models parameters, we perform the Maximum Likelihood Estimation for each model. Under the same set of historical data, the MLE testing will be performed on each model separately. The historical data of Hang Seng index return ranges from 1-Jan-87 to 20-Dec-02, which including both 87, 97 market crash –extremely high volatility period, and the recent low volatility period of 2002.

--The actual implementation of the MLE is on the Excel spreadsheet file: ***HSI_GARCHs.xls***

Step1: Take historical data –the daily closing index level from 1-Jan-87 to 20-Dec-02.

Step2: Calculate daily return and the volatility for each day by iterative formula (different model will have different iterative formula).

Step3: Calculate the log-likelihood of each day, and then take the summation for whole set of log-likelihood, L

(Note: The value of log-likelihood for each day is $-(1/2)*\log(2\pi) - (1/2)*\log(\sigma^2) - (1/2)*(\varepsilon/\sigma)^2$)

Step4: Using Excel solver to maximize the L value by varying the models parameters, (alpha, beta, gamma, omega)

Step5: The resulting parameters are the optimal values.

---GJR-GARCH is the best choice here, which gives the highest log-likelihood value, L.

---In using these models parameters to simulate index level, we are assuming that the model is stationary with time.

i.e. The index will perform the same characteristic in the future time.

GJR-GARCH model		**parameters**	
omega =	0.002216	**log_L =**	6339.878
alpha =	0.035261		
beta =	0.904235		
gamma =	0.09863		

Details on HSI index simulation and Guaranteed Fund pricing:

In this Monte Carlo Simulation spreadsheet, ***GJR_GFUND.xls****, the 4 years period is included. The GJR-GARCH model is used to simulate the* daily return. $\varepsilon(i)$ = **N~[(interest rate -dividend)*dt,** σ**(i)*√dt]**, *where* σ**(i)** *is the daily volatility by the GJR_GRACH iterative formula with a certain initial volatility* σ*. Use these daily returns, the daily index level I can be calculated. We assume that there are 252 trading day in one year, so totally 252 x 4 = 1008 trading days in 4 year period.*

By the end of each month (the yellow cells on the spreadsheet), the index level is marked down and the up-side return is recorded,

Max (I (n)/I (0) - 1, 0)

Then, taking average on all the 48 monthly upside returns and get the value of C, the Final monthly average positive growth % of HSI

$$C = [\Sigma_{n=1 \text{ to } 48} \{ Max (I(n)/I(0) - 1, 0) \}] / 48$$

So, for the X % level Guaranteed Fund with participation rate k, the ***Final Payoff at maturity*** (at the end of the 4th years)

$$\textit{Final Payoff} = P * X\% + P * k * C$$

Taking the PV of the final payoff (Final Payoff * exp(-r*T)), and run the simulation for thousands times, the value of the Guaranteed Fund is obtained as the mean of this PV (Final payoff). *This procedure is under the assumption that the interest rate during the 4 years period is constant*.

Simulation Results of GJR_GFUND.xls :

The GJR-GARCH modeling parameters (omega, alpha, beta gamma) are given from pervious section.
Interest Rate: 5%
Dividend: 3%
Initial Volatility: 30% (a good reference is the implied volatility of the market traded option /or OTC products with similar maturity on the same underlying index)
Period: 4 years
Principal investment: US$100

90% Guaranteed:	value of Bond part -->	73.685	
Participation:	Fund value at issuing date:	Value of option:	Bank Profit: (US$100 Fund value)
85%	86.385	12.699	13.615
92%	87.43	13.744	12.57
100%	88.625	14.939	11.375

100% Guaranteed:	value of Bond part -->	81.873	
Participation:	Fund value at issuing date:	Value of option	Bank Profit: (US$100 Fund value)
40%	87.849	5.9759	12.151
48%	89.044	7.1709	10.956
55%	90.09	8.2169	9.91

110% Guaranteed:	value of Bond part -->	90.0603	

Participation:	*Fund value at issuing date:*	*Value of option:*	*Bank Profit: (US$100 Fund value)*
5%	*90.8074*	*0.7470*	*9.1926*
10%	*91.554*	*1.4936*	*8.446*
15%	*92.301*	*2.2406*	*7.699*

Section C:
Analysis on Profitability:

Apart from the 1.25% p.a. management fee, it is obviously that the offering price US$100 is much higher than the fair value of the Guaranteed Fund (according to our simulation results). This gives another source of profit for the bank.
From the pervious section, we have seen that the Guaranteed Fund can be divided into two parts—the Bond part and the Asian type Call option part. For each US$100 principal investment, the higher guaranteed level will lead to more capital investment into the bond.
90% Guaranteed Level → invest US$73.686 into bond
100% Guaranteed Level → invest US$81.873 into bond
110% Guaranteed Level → invest US$90.06 into bond
The higher guaranteed level also implied that there is less money to invest into the Asian call option, hence the lower participation rate.
Moreover, under the GJR-GRACH model, we can observe from the simulation that the value of the Asian call is only about US$14.94 (observe in 100% participation rate case). In other words, *the bank can have a great flexibility to choose the participation rate so as to capture a "big profit"*. The relationship of participation rate and the bank profit is shown in the above table. *(Bank Profit = US$100 - Fund value)*

Note that the amount of capital for bond investment also depends on the current interest rate. In high interest condition, the bond investment amount should be smaller, and hence higher profit potential for the bank. However, in such a high interest environment, this kind of capital guaranteed product may not be so attractive to retail investors.

Why the bank does not fix an exact value of participation rate until fund launch date?
During the promotion period, a range of participation rate is given for each guaranteed fund.
The bank will set the actual participation rate until the launch date. They do not give out the exact value of participation rate in early promotion period, so as to have an extra flexibility to adapt the market fluctuation. For example, if during the 2 days before the fund launching, the market becomes highly volatile. The long-term volatility is not much affected, and so is the Hang Seng Capital Guaranteed Fund price. However, the implied volatility in short-term traded option marketwill raise dramatically. If there are only short-term options available in the market for hedging, the cost of hedging the Hang Seng Index-linked option of the guaranteed fund will be increased. To maintain high profit in launching this Fund, issuer will lower the participation rate to compensate for the high hedging cost. Conversely, if the market has been steady before the fund launching, issuer will set a higher participation rate to make the Fund more attractive to investors whilst capturing a decent profit.

***** Comparing the Hang Seng Capital Guaranteed Fund Price under stochastic volatility model (GJR-GARCH) and constant volatility model (BS model)***
--We use GJR_GFUND.xls & BS_GFUND.xls to simulate the Fund price with different initial volatility σ.
(BS_GFUND.xls is a Monte Carlo simulation spreadsheet on the Guaranteed Fund with constant volatility.)

Take 90% Guaranteed Fund 85% participation rate as an example:

Simulation on The Hang Seng Capital Guaranteed Fund with both asset pricing model:

Guaranteed level:	**90%**	***Fund price:***	***Sigma at issuing date:***			
Participation rate:	**85%**		***50%***	***30%***	***25%***	***15%***
Interest rate:	**5%**	***GJR Model***	***87.70***	***86.38***	***86.12***	***85.7***
Dividend:	**3%**	***Constant volatility Model***	***93.39***	***86.4***	***84.58***	***80.92***

The Fund price under BS type and GJR-GARCH will be very similar only when the initial volatility is about 30%. If we use 50% as initial volatility at issuing date, BS model will give a much higher price than GJR_GARCH. On the other hand, if we use 15% volatility, BS model will give a lower price.

This finding is matched with the theory that GARCHs model has a mean reverting feature.
If volatility is 50% initially, the GARCH model will 'revert' it to a mean around 30%
If volatility is 15% initially, the GARCH model will pull it up to 30% also. This means that BS model will overvalue in high volatility environment, and undervalue in low volatility case.--This demonstrates the importance of using a GARCH model in this long-term derivative product.

Section D:
Hedging and Replication

The Guaranteed Funds will invest in Hang Seng index-linked Notes whose return at maturity will match that of the Funds respectively. In effect, the Notes will comprise zero coupon instruments (4-year zero coupon bond) and Hang Seng Index-linked option. (Asian type Call option which take the 48-average monthly positive return as payoff).

By separating this guaranteed fund into these two parts, we can do the following analysis on hedging and replication.

Zero coupon instruments*:*

Method 1: Since the base currency of the Fund is US$, the bank can invest the appropriate amount of capital (depends on guaranteed level) into the US zero-coupon bond with 4-year maturity. The only job for the bank is to buy the T-bill and hold it until maturity. As the interest rate is locked at the beginning, there is no need to manage the interest rate risk.

Method 2: This Fund is offered and structured by Hang Seng Bank. As a commercial bank in HK, the major business is capital lending and borrowing. The bond part of this guaranteed fund *becomes one of the cheap funding sources*. (just like the retail certificate of deposit, CD) For example, the bank may give 5% guaranteed return to the fund investor and lend out the capital to SME by adding a spread. In this case, the bank may need to do more on credit risk management with the SME borrower.

<u>Hang Seng Index-linked option</u>:

This is an exotic option with 48 months average positive return as payoff.

<u>Method 1</u>: Hedge by buying a similar product in OTC market The specification of this OTC product should be exactly the same as the Asian Call option part with payoff $C = [\Sigma_{n=1 \text{ to } 48} \{ Max (I(n)/I(0) - 1 , 0) \}] /48$ and 4 year maturity.

<u>Method 2:</u> Hedge with a sequence of plain vanilla call with strike at the initial index level I(0).
At the beginning of each month, the bank can buy the one-month plain vanilla call option. (The amount of investment for each month is only 1/48 of a Hang Seng Asian call). The payoff of this call can hedge against the index linked option part during that month. *However, it may not be an effective hedging method, since the total hedging cost can be very large.*

<u>Method 3</u>: Dynamic delta hedging method with *Hang Seng Index Future*. Just like the plain vanilla option, the index-linked option can be hedged by dynamic delta hedging. The delta value can be calculated by "bumping the spot" in Monte Carlo simulation.

Theoretically, the Hang Seng Index Future, the 33 component stocks of Hang Seng Index, and the Exchange Traded Fund—Tracker Fund can all be used for delta hedging. However, in practice, *only Hang Seng Index Future is the cost effective method due to its high liquidity and very small transaction cost.* In order to minimize transaction cost in frequent hedging, it is possible to have a dynamic delta hedging with filter of 1% index movement.

The bank may also need to consider the hedging of vega risk on this index-linked option. However, since its payoff is taking an average over 48 months. The risk to the change in volatility will be smaller than plain vanilla option. The effect of vega risk can be neglected.